

**Title:****Socratic AI Tutoring in Primary School Mathematics: A Case Study on the Development of Problem-Solving and Digital Competence According to DigComp 2.2**

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**Abstract**

The integration of digital technologies is profoundly transforming traditional educational models. This study investigates the effectiveness of a digital Socratic tutoring approach in enhancing mathematical problem-solving skills in primary school students, using the European DigComp 2.2 framework as a reference. Through a qualitative case study conducted in a fifth-grade classroom, the interaction between students and an AI tutor during math activities was analyzed. The preliminary results are promising. They show that Socratic questioning fosters both mathematical and digital competencies, aligning with recent research on AI-mediated reflective learning. The findings support the potential for scalable pedagogical models that integrate critical thinking, autonomy, and cognitive engagement, addressing systemic educational disparities and promoting equity. It is important to note that this study is a small-scale case study with a limited sample size and timeframe. Future research with larger and more diverse cohorts is needed to validate these findings.

**Keywords:**

Socratic AI tutor, DigComp 2.2, critical thinking, primary education, mathematical problem-solving, digital competence, chatbot, generative AI.

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**1. Introduction**

The educational demands of the 21st century emphasize advanced literacy, critical thinking, and problem-solving, as outlined by European [1], [2] and the updated Digital Competence Framework for Citizens [3]. Despite progress, Italy continues to face significant educational disparities. National assessments (INVALSI, 2024) reveal challenges in reading comprehension and mathematics starting as early as primary school [4].

Addressing these challenges requires pedagogical interventions that foster reflective and analytical reasoning. Socratic tutoring, delivered through generative AI, represents a promising approach. Unlike conventional chatbots, Socratic agents pose thought-provoking questions to guide learners in self-directed reasoning [5]. This aligns with the cognitive principles of Vygotsky's scaffolding and Bruner's spiral curriculum.

This study aims to explore the pedagogical impact of a Socratic AI tutor on mathematical comprehension and digital literacy, focusing on DigComp 2.2 areas: information navigation (1.1), evaluation (1.3), and problem-solving (5.3).

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**2. Methodology****2.1 Context and Participants**

The study employed a qualitative design involving 18 students (aged 10–11) from a fifth-grade class at "Antonio Nuzzo" Primary School (Rome, Italy), within a culturally and linguistically diverse environment. The class presented heterogeneity in academic performance, with cases of high achievement as well as risk of disengagement. A limitation of this study is the absence of a comparison group (e.g., teacher-led Socratic questioning or traditional problem-solving). Future studies could include such comparisons to isolate the specific impact of the AI tutor.

**2.2 AI Tools and Instructional Design**

The intervention utilized generative AI chatbots (Gemini and DeepSeek), accessed solely via the teacher's device to ensure GDPR compliance. The AI agents were prompt-engineered to function as Socratic tutors, guiding students through structured problem-solving via targeted questioning. The AI tutor was designed using the following Socratic questioning framework:

- Clarification Questions: "What do you think is the key information here?"
- Probing Assumptions: "Why did you choose this strategy?"
- Alternative Perspectives: "Is there another way to approach this problem?"

Example sequences and teacher's actions are provided in the annex.

### 2.3 Learning Approach

The activities combined Problem-Based Learning and Cooperative Learning. Students worked individually and in small groups. The mathematical tasks were rephrased into elaborate narrative forms (e.g., "Uncle Bepi's vineyard"), using complex vocabulary and syntax to encourage critical text analysis and reflection.

Each interaction unfolded in three guided phases:

1. Identification of key data;
2. Planning of a general solution (without numerical computation);
3. Execution of operations.

The AI tutor's questions were structured according to established Socratic taxonomies [6],[7], encouraging cognitive engagement and metacognition.

### 2.4 Data analysis

pre-and post-intervention assessments were scored using the rubric usually used for evaluation. Growth was measured by comparing individual student scores before and after the intervention. Teacher observations were coded for engagement and metacognitive behaviours using DigComp 2.2 indicators.

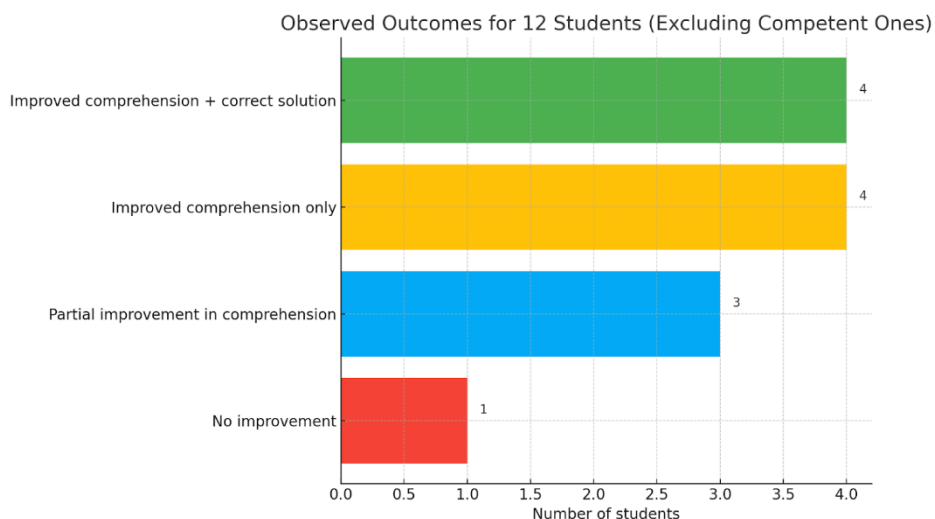
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## 3. Results

The activity targeted DigComp 2.2 competencies:

- **1.1 Information and data literacy:** Students demonstrated improved skills in identifying and extracting relevant numerical data from complex texts.
- **5.3 Problem-solving:** 78% of interactions reflected growing autonomy in devising and validating solution strategies.
- **1.3 Evaluation:** Students corrected initial errors through iterative AI-mediated dialogue, reflecting increased critical assessment skills.

Of the 18 participants (excluding those already proficient), 8 showed marked improvement in text comprehension; 4 of these also improved mathematically. Three showed moderate gains, and one exhibited no appreciable progress, highlighting the need for more personalized support.



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## 4. Discussion

These findings confirm the potential of AI-mediated Socratic dialogue in fostering System 2 thinking—deliberate, reflective cognition—as theorized [8]. Rather than replacing instruction, the AI served as a "more knowledgeable other" (Vygotsky, 1980), providing scaffolded cognitive guidance.

Our results align with recent experiments using Socratic LLMs [5] and chatbot-mediated research development tools [9], which emphasize questioning over direct instruction to stimulate analytical

thought. The ability of AI to act as a dialogic partner, rather than a solution provider, differentiates this approach from traditional intelligent tutoring systems.

Moreover, the intervention promoted transdisciplinary learning: students integrated linguistic interpretation, logical reasoning, and numerical calculation, reflecting the holistic nature of real-world problem-solving.

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## 5. Conclusion

Recent studies (e.g., Favero et al., 2024; Degen, 2025) have demonstrated the role of Socratic AI tutors in fostering critical thinking. This work builds on these findings by focusing on primary education and integrating the DigComp 2.2 framework. It would explore how the Socratic digital tutoring, when integrated into authentic educational contexts, can significantly support the development of both mathematical reasoning and digital literacy in primary education. The structured, question-driven dialogue promoted metacognitive regulation, autonomy, and cross-disciplinary learning.

By targeting DigComp 2.2 indicators (1.1, 1.3, 5.3), the intervention demonstrate the potential of a scalable, evidence-based model for future STEM education in digitally mediated settings.

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## 6. Future Directions

To validate these promising results, future studies should:

- Expand the sample size and demographic scope.
- Introduce mathematically complex, open-ended problems.
- Investigate longitudinal outcomes and skill transfer across subjects.
- Compare local LLM-based tutors with cloud-based models for data privacy and equity.

Integrating Socratic AI tutors in flipped classrooms or inclusive educational programs may significantly contribute to more equitable and student-centered mathematics education.

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## ANNEX

### PROBLEM TEXT EXAMPLE

Mr. Bepi, a winemaker of renowned experience, harvested a large quantity of grapes, equal to 900 quintals.

Of this harvest,  $\frac{3}{5}$  are made up of the Rondinella variety, while  $\frac{2}{3}$  of the remaining part are represented by the fine Garganega variety, the remaining fraction is made up of the Teroldega grape. Mr. Bepi decides to market the Rondinella grape at the price of € 70 per quintal and the Garganega grape at a price that corresponds to  $\frac{7}{10}$  of that of the Rondinella.

### PROMPT

“You are a Socratic tutor and you have to support 11-year-old pupils in solving the following problem, involving a phase of identifying the data, planning the solution with mathematical formulation without numbers and then the operations necessary for the solution by stimulating reasoning:” Mr Bepi, a wine-grower of renowned experience, has harvested a large quantity of grapes, amounting to 900 quintals. Of this harvest,  $\frac{3}{5}$  are of the Rondinella variety, while  $\frac{2}{3}$  of the remainder are Garganega grapes, and the remaining fraction is represented by the Terodola variety. Mr Bepi decides to market the Rondinella grapes at a price of 70 euros per quintal, and the Garganega grapes at a price equal to  $\frac{7}{10}$  of the value of the Rondinella. Determine the income from the sale of the two grape varieties, and calculate the quantity of grapes Mr Bepi will keep for the production of his wine.”

### SOCRATIC DIALOGUE EXAMPLE

Characters:

- **User (U):** The one who is trying to solve the problem.
- **Guide (G):** The one who asks the Socratic questions.

#### Dialogue:

G: *Well, User, reading this issue, what's the first thing that catches your eye? What is the fundamental information that we are given at the beginning?*

U: Mr. Bepi has harvested 900 quintals of grapes in total.

G: *Exactly. And then, the text begins to describe how this total amount is divided. What is the first subdivision that is mentioned?*

U: You say that the 53 are of the Rondinella variety.

G: *What does this fraction mean in terms of quantity compared to the total? How can we understand how many quintals of Rondinella Mr. Bepi has collected?*

U: We should calculate  $\frac{3}{5}$  of 900 quintals.

G: *Great. And do you know how to do this calculation? What is the mathematical operation needed?*

U: We must multiply  $\frac{3}{5}$  by 900.

G: *Feel free to proceed with this calculation. How much does it make?*

U: ... (User performs the calculation) ... It makes 540 quintals.

G: *So, we understood that there are 540 quintals of Rondinella grapes. The text then goes on to describe the remaining part of the grapes. How many grapes are left after considering the Rondinella?*

U: If the total was 900 and 540 are Rondinella, then  $900 - 540 = 360$  quintals remain.

G: *Perfect. And what does the text tell us about these 360 quintals? How are they further divided?*

U: He says that the  $\frac{2}{3}$  of the remaining part are of the Garganega variety.

G: *Once again, what do we have to do to understand how many quintals of Garganega there are?*

U: We have to calculate the  $\frac{2}{3}$  of 360 quintals.

G: *How do we do it?*

U: Let's multiply  $\frac{2}{3}$  by 360.

G: *Calculate as well. What is the result?*

U: ... (User performs the calculation) ... It makes 240 quintals.

G: *So, we have 240 quintals of Garganega. The text also mentions another variety, Teroldega. How can we determine the amount of Teroldega grapes?*

U: It says it's the residual fraction. So, if we had 360 quintals after removing the Rondinella, and 240 are Garganega, the Teroldega will be  $360 - 240 = 120$  quintals.

G: *Exactly. We have now determined the quantity of each variety. The last part of the text concerns selling prices. What is the price per quintal of Rondinella grapes?*

U: €70 per quintal.

G: *And what is the price of Garganega grapes? Pay attention to how it is described.*

U: You say that the price corresponds to the  $\frac{7}{10}$  of that of the Rondinella.

G: *What do we have to do to find the price of Garganega?*

U: We have to calculate the  $\frac{7}{10}$  of 70€.

G: *And how do you do this calculation?*

U: Let's divide 70 by 10 and then multiply by 7.

G: *Calculate the price of Garganega. How much does it make?*

U: ... (User performs the calculation) ... That's €49 per quintal.

G: *Great. Now, looking back at the problem as a whole, what was the possible purpose of all this information? Was there an implicit or explicit question that the text might want us to solve?*

U: The text does not ask a direct question, but we might want to calculate Mr. Bepi's total revenue from the sale of these grapes.

G: *If we wanted to calculate total revenue, what steps should we follow, based on what we have understood so far?*

U: We should multiply the quantity of each type of grape by its price per quintal and then add up the results.

G: *Perfect. You now have a clear understanding of the information provided and have identified the path to calculate a possible interesting outcome. This step-by-step analysis process, answering targeted questions, helped you break down potentially problematic text into more manageable parts and understand the relationships between different pieces of information.*

## **TEACHER'S ACTION**

### **Preparation Phase:**

- Designed narrative-based math problems (e.g., "Uncle Bepi's vineyard") to contextualize learning.
- Configured the AI tutor with Socratic questioning prompts.
- Explained the chatbot's purpose and evaluation criteria to students.
- Organized students into small, heterogeneous groups.

### **Implementation Phase:**

- Introduced the AI tutor via the teacher's device (GDPR-compliant access).
- Monitored group work, intervening to clarify doubts and scaffold thinking.
- Posed metacognitive questions (e.g., "Why did you choose this strategy?") to deepen reasoning.
- Guided the three-step problem-solving process:
  - Data identification
  - Strategy planning
  - Numerical computation
- Recorded observations on student engagement, collaboration, and metacognitive behaviours.

### **Assessment & Reflection Phase:**

- Conducted a triangular evaluation combining:
  - Student self-assessment
  - Peer feedback
  - Teacher evaluation using a DigComp 2.2-aligned rubric (focusing on digital and math competencies).
- Facilitated guided reflection sessions to analyze problem-solving strategies.